Mathematik-Intensivierung * Jahrgangsstufe 9
Potenzen mit rationalen Exponenten

Definition der allgemeinen Wurzel

$\sqrt[n]{a} = a^{\frac{1}{n}}$ und $\sqrt[n]{a^m} = a^{\frac{m}{n}}$ für $a \in \mathbb{R}_o^+$, $n \in \{2, 3, 4, 5, ...\}$, $m \in \mathbb{Z}$

Potenzgesetze:

(I) $a^{\frac{m}{n}} \cdot a^{\frac{s}{t}} = a^{\frac{m}{n}+\frac{s}{t}}$ (II) $a^{\frac{m}{n}} : a^{\frac{s}{t}} = a^{\frac{m}{n}-\frac{s}{t}}$ (III) $\left(a^{\frac{m}{n}}\right)^s = a^{\frac{m \cdot s}{n}}$

(IV) $a^{\frac{m}{n}} \cdot b^{\frac{m}{n}} = (ab)^{\frac{m}{n}}$ (V) $a^{\frac{m}{n}} : b^{\frac{m}{n}} = (a:b)^{\frac{m}{n}}$

Aufgaben:

1. Vereinfache und stelle den Term als Potenz mit rationalem Exponenten dar.

a) $\sqrt[4]{144}$ b) $\sqrt[6]{625}$ c) $\sqrt[9]{125}$ d) $\sqrt[10]{32}$

2. Vereinfache und stelle den Term als Wurzel mit möglichst kleinem Wurzelexponenten dar.

a) $8^{\frac{1}{6}}$ b) $25^{\frac{1}{8}}$ c) $27^{\frac{1}{9}}$ d) $16^{\frac{1}{12}}$

e) $81^{\frac{1}{10}}$ f) $81^{\frac{1}{20}}$ g) $216^{\frac{1}{6}}$ h) $32^{\frac{1}{15}}$

3. Vereinfache durch teilweises Radizieren und schreibe den Term als Wurzel.

a) $\sqrt[4]{512}$ b) $81^{\frac{1}{3}}$ c) $\sqrt[3]{250}$ d) $144^{\frac{3}{5}}$

e) $\sqrt[3]{49^2}$ f) $24^{\frac{2}{3}}$ g) $\sqrt[5]{72^2}$ h) $8^{\frac{3}{4}}$

4. Mache den Nenner rational und schreibe den Term als Wurzel.

a) $\sqrt[4]{\frac{1}{8}}$ b) $6{,}75^{\frac{1}{3}}$ c) $\frac{2}{\sqrt[3]{25}}$ d) $0{,}25^{\frac{3}{5}}$

e) $\frac{2}{\sqrt[4]{9}}$ f) $49^{-\frac{1}{3}}$ g) $\frac{6}{\sqrt[6]{27}}$ h) $8^{-\frac{3}{4}}$

5. Bestimme die Lösungsmenge und gib die Lösung als möglichst weit vereinfachten Wurzelterm an.

a) $2 \cdot x^3 = 32$ b) $0{,}2 \cdot x^3 = -32$ c) $2 - 3x^4 = 5$

d) $2 \cdot x^3 - 4 = 5$ e) $0{,}25 \cdot x^6 + 4 = 4^{1,5}$ f) $0{,}5 \cdot (x-1)^3 - 4 = 12$

g) $8 \cdot x^{-3} = 5$ h) $2 \cdot x^{-5} - 5 = 3^3$ i) $0{,}5 \cdot x^{-5} + 11 = 3$

j) $2 \cdot x^{\frac{1}{3}} + 4 = 5$ k) $0{,}5 \cdot x^{\frac{2}{3}} + 3 = 5$ l) $(x-2)^{\frac{1}{4}} - 3 = 5$

6. Vereinfache den Term und stelle ihn in Potenz- und Wurzelschreibweise dar.

a) $3^{0,5} \cdot 9^{0,75}$

b) $4^{\frac{1}{5}} \cdot 2^{\frac{1}{10}} : 8^{\frac{1}{4}}$

c) $\sqrt[4]{27} \cdot \sqrt[5]{9} \cdot \sqrt[20]{27}$

d) $\sqrt[3]{4} : \sqrt[3]{36}$

e) $\sqrt[3]{\frac{4}{5}} : \sqrt[6]{25}$

f) $\sqrt[5]{\frac{8}{9}} : \sqrt[5]{\frac{27}{128}}$

g) $\sqrt[3]{4 \cdot \sqrt[4]{8}}$

h) $\sqrt[5]{9 \cdot \sqrt[3]{3 \cdot \sqrt{27}}}$

i) $\sqrt[6]{6 \cdot \sqrt[4]{6 \cdot \sqrt[3]{6}}}$

7. Vereinfache den Term und stelle ihn in Potenz- und Wurzelschreibweise dar.

a) $\sqrt[4]{a^2 \cdot \sqrt[3]{a^2 \cdot \sqrt{a}}}$

b) $\sqrt[3]{\frac{a^2}{b} \cdot \sqrt[2]{\frac{a^3}{b}} \cdot \sqrt[4]{\frac{b^3}{a^5}}}$

c) $\sqrt[5]{c^4 \cdot \sqrt[3]{c^2}} \cdot \sqrt{c \cdot \sqrt[4]{c^3}} : \sqrt[24]{c^{41}}$

d) $\sqrt[4]{a \cdot \sqrt[6]{ab^2}} \cdot \sqrt[3]{\sqrt[8]{a^{11}b^{-1}}} \cdot \sqrt{b^2 \cdot \sqrt[4]{a^2}}$

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1. a) $\sqrt[4]{144} = \sqrt[4]{2^4 \cdot 3^2} = 2 \cdot 3^{\frac{2}{4}} = 2 \cdot 3^{\frac{1}{2}} = 2 \cdot \sqrt{3}$ b) $\sqrt[6]{625} = \sqrt[6]{5^4} = 5^{\frac{4}{6}} = 5^{\frac{2}{3}} = \sqrt[3]{25}$

c) $\sqrt[9]{125} = \sqrt[9]{5^3} = 5^{\frac{3}{9}} = 5^{\frac{1}{3}} = \sqrt[3]{5}$ d) $\sqrt[10]{32} = \sqrt[10]{2^5} = 2^{\frac{5}{10}} = 2^{\frac{1}{2}} = \sqrt{2}$

2. a) $8^{\frac{1}{6}} = \left(2^3\right)^{\frac{1}{6}} = 2^{\frac{1}{2}} = \sqrt{2}$ b) $25^{\frac{1}{8}} = (5^2)^{\frac{1}{8}} = 5^{\frac{1}{4}} = \sqrt[4]{5}$

c) $27^{\frac{1}{9}} = (3^3)^{\frac{1}{9}} = 3^{\frac{1}{3}} = \sqrt[3]{3}$ d) $16^{\frac{1}{12}} = (2^4)^{\frac{1}{12}} = 2^{\frac{1}{3}} = \sqrt[3]{2}$

e) $81^{\frac{1}{10}} = (3^4)^{\frac{1}{10}} = 3^{\frac{2}{5}} = \sqrt[5]{9}$ f) $81^{\frac{1}{20}} = (3^4)^{\frac{1}{20}} = 3^{\frac{1}{5}} = \sqrt[5]{3}$

g) $216^{\frac{1}{6}} = (6^3)^{\frac{1}{6}} = 6^{\frac{1}{2}} = \sqrt{6}$ h) $32^{\frac{1}{15}} = (2^5)^{\frac{1}{15}} = 2^{\frac{1}{3}} = \sqrt[3]{2}$

3. a) $\sqrt[4]{512} = \sqrt[4]{2^9} = 4 \cdot \sqrt[4]{2}$ b) $81^{\frac{1}{3}} = (3^4)^{\frac{1}{3}} = 3^{\frac{4}{3}} = 3^{1+\frac{1}{3}} = 3 \cdot 3^{\frac{1}{3}} = 3 \cdot \sqrt[3]{3}$

c) $\sqrt[3]{250} = \sqrt[3]{2 \cdot 5^3} = 5 \cdot \sqrt[3]{2}$ d) $144^{\frac{3}{5}} = (2^4 \cdot 3^2)^{\frac{3}{5}} = 2^{\frac{12}{5}} \cdot 3^{\frac{6}{5}} = 2^2\, 2^{\frac{2}{5}} \cdot 3^1 \cdot 3^{\frac{1}{5}} = 12 \cdot \sqrt[5]{12}$

e) $\sqrt[3]{49^2} = \sqrt[3]{7^4} = 7 \cdot \sqrt[3]{7}$ f) $24^{\frac{2}{3}} = (2^3 \cdot 3)^{\frac{2}{3}} = 2^{\frac{2 \cdot 3}{3}} \cdot 3^{\frac{2}{3}} = 4 \cdot \sqrt[3]{9}$

g) $\sqrt[5]{72^2} = \sqrt[5]{2^6 \cdot 3^4} = 2 \cdot \sqrt[5]{2 \cdot 3^4} = 2 \cdot \sqrt[5]{162}$ h) $8^{\frac{3}{4}} = (2^3)^{\frac{3}{4}} = 2^{\frac{9}{4}} = 2^{2+\frac{1}{4}} = 4 \cdot \sqrt[4]{2}$

4. a) $\sqrt[4]{\frac{1}{8}} = \sqrt[4]{\frac{1 \cdot 2}{2^3 \cdot 2}} = \frac{\sqrt[4]{2}}{2}$ b) $6{,}75^{\frac{1}{3}} = \sqrt[3]{\frac{27}{4}} = \sqrt[3]{\frac{3^3 \cdot 2}{2^2 \cdot 2}} = \frac{3 \cdot \sqrt[3]{2}}{2}$

c) $\frac{2}{\sqrt[3]{25}} = \frac{2 \cdot \sqrt[3]{5}}{\sqrt[3]{5^2} \cdot \sqrt[3]{5}} = \frac{2 \cdot \sqrt[3]{5}}{5}$ d) $0{,}25^{\frac{3}{5}} = \sqrt[5]{\left(\frac{1}{4}\right)^3} = \sqrt[5]{\left(\frac{1}{2^2}\right)^3} = \sqrt[5]{\frac{1 \cdot 2^4}{2^6 \cdot 2^4}} = \frac{\sqrt[5]{16}}{4}$

e) $\frac{2}{\sqrt[4]{9}} = \frac{2 \cdot \sqrt[4]{3^2}}{\sqrt[4]{3^2} \cdot \sqrt[4]{3^2}} = \frac{2 \cdot \sqrt[4]{3^2}}{3}$ f) $49^{-\frac{1}{3}} = \frac{1}{7^{\frac{2}{3}}} = \frac{1 \cdot 7^{\frac{1}{3}}}{7^{\frac{2}{3}} \cdot 7^{\frac{1}{3}}} = \frac{7^{\frac{1}{3}}}{7^1} = \frac{\sqrt[3]{7}}{7}$

g) $\frac{6}{\sqrt[6]{27}} = \frac{6}{3^{\frac{3}{6}}} = \frac{6 \cdot 3^{\frac{1}{2}}}{3^{\frac{1}{2}} \cdot 3^{\frac{1}{2}}} = 2 \cdot \sqrt{3}$ h) $8^{-\frac{3}{4}} = \frac{1}{(2^3)^{\frac{3}{4}}} = \frac{1}{2^{\frac{9}{4}}} = \frac{1 \cdot 2^{\frac{3}{4}}}{2^{\frac{9}{4}} \cdot 2^{\frac{3}{4}}} = \frac{2^{\frac{3}{4}}}{2^3} = \frac{\sqrt[4]{8}}{8}$

5. a) $2 \cdot x^3 = 32 \Leftrightarrow x^3 = 16 \Leftrightarrow x = \sqrt[3]{2^4} = 2 \cdot \sqrt[3]{2}$

b) $0{,}2 \cdot x^3 = -32 \Leftrightarrow x^3 = -160 \Leftrightarrow x = -\sqrt[3]{160} \Leftrightarrow x = -2 \cdot \sqrt[3]{20}$

c) $2 - 3x^4 = 5 \Leftrightarrow 3x^4 = -3 \Leftrightarrow x^4 = -1$ keine Lösung!

d) $2 \cdot x^3 - 4 = 5 \Leftrightarrow 2 \cdot x^3 = 9 \Leftrightarrow x^3 = \frac{9}{2} \Leftrightarrow x = \sqrt[3]{\frac{9 \cdot 4}{2 \cdot 4}} = \frac{\sqrt[3]{36}}{2}$

e) $0{,}25 \cdot x^6 + 4 = 4^{1{,}5} \Leftrightarrow x^6 + 16 = 4 \cdot 4^{1{,}5} \Leftrightarrow x^6 = 4 \cdot 8 - 16 \Leftrightarrow x^6 = 16 \Leftrightarrow$

$x_{1/2} = \pm \left(2^4\right)^{\frac{1}{6}} \Leftrightarrow x_{1/2} = \pm 2^{\frac{2}{3}} \Leftrightarrow x_{1/2} = \pm \frac{\sqrt[3]{2}}{2}$

f) $0{,}5 \cdot (x-1)^3 - 4 = 12 \Leftrightarrow (x-1)^3 = 16 \cdot 2 \Leftrightarrow x - 1 = \sqrt[3]{32} \Leftrightarrow x = 1 + 2 \cdot \sqrt[3]{4}$

g) $8 \cdot x^{-3} = 5 \Leftrightarrow \frac{8}{5} = x^3 \Leftrightarrow x = \sqrt[3]{\frac{8}{5}} = \sqrt[3]{\frac{2^3 \cdot 5^2}{5 \cdot 5^2}} = \frac{2 \cdot \sqrt[3]{25}}{5}$

h) $2\cdot x^{-5}-5=3^3 \Leftrightarrow 2\cdot x^{-5}=32 \Leftrightarrow \frac{1}{x^5}=16 \Leftrightarrow x^5=\frac{1}{2^4} \Leftrightarrow x=\sqrt[5]{\frac{1\cdot 2}{2^4\cdot 2}}=\frac{\sqrt[5]{2}}{2}$

i) $0{,}5\cdot x^{-5}+11=3 \Leftrightarrow \frac{1}{2\cdot x^5}=-8 \Leftrightarrow -\frac{1}{2\cdot 8}=x^5 \Leftrightarrow x^5=-\frac{2}{2^5} \Leftrightarrow x=-\frac{\sqrt[5]{2}}{2}$

j) $2\cdot x^{\frac13}+4=5 \Leftrightarrow x^{\frac13}=\frac12 \Leftrightarrow x=\left(\frac12\right)^3=\frac18$

k) $0{,}5\cdot x^{\frac23}+3=5 \Leftrightarrow x^{\frac23}=2\cdot 2 \Leftrightarrow x=(4)^{\frac32}=2^3=8$

l) $(x-2)^{\frac14}-3=5 \Leftrightarrow (x-2)^{\frac14}=8 \Leftrightarrow x-2=8^4 \Leftrightarrow x=4098$

6. a) $3^{0,5}\cdot 9^{0,75}=3^{\frac12}\cdot 3^{\frac32}=3^2=9$

b) $4^{\frac15}\cdot 2^{\frac1{10}}:8^{\frac14}=2^{\frac25}\cdot 2^{\frac1{10}}:2^{\frac34}=2^{\frac25+\frac1{10}-\frac34}=2^{-\frac14}=\frac{1}{\sqrt[4]{2}}=\frac{\sqrt[4]{8}}{2}$

c) $\sqrt[4]{27}\cdot\sqrt[5]{9}\cdot\sqrt[20]{27}=3^{\frac34}\cdot 3^{\frac25}\cdot 3^{\frac3{20}}=3^{\frac34+\frac25+\frac3{20}}=3^{\frac{13}{10}}=3\cdot 3^{\frac3{10}}=3\cdot\sqrt[10]{27}$

d) $\sqrt[3]{4}:\sqrt[3]{36}=2^{\frac23}:6^{\frac23}=2^{\frac23}:6^{\frac23}=\frac{2^{\frac23}\cdot 6^{\frac13}}{6^{\frac23}\cdot 6^{\frac13}}=\frac{4^{\frac13}\cdot 6^{\frac13}}{6}=\frac{(2\cdot 2\cdot 2\cdot 3)^{\frac13}}{6}=\frac{2\cdot 3^{\frac13}}{6}=\frac{1\cdot 3^{\frac13}}{3}=\frac{\sqrt[3]{3}}{3}$

e) $\sqrt[3]{\frac45}:\sqrt[6]{25}=\frac{2^{\frac23}}{5^{\frac13}}:(5^2)^{\frac16}=\frac{2^{\frac23}}{5^{\frac13}\cdot 5^{\frac26}}=\frac{2^{\frac23}}{5^{\frac23}}=\frac{2^{\frac23}\cdot 5^{\frac13}}{5^{\frac23}\cdot 5^{\frac13}}=\frac{4^{\frac13}\cdot 5^{\frac13}}{5}=\frac{20^{\frac13}}{5}=\frac{\sqrt[3]{20}}{5}$

f) $\sqrt[5]{\frac89}:\sqrt[5]{\frac{27}{128}}=\frac{2^{\frac35}}{3^{\frac25}}:\frac{3^{\frac35}}{2^{\frac75}}=\frac{2^{\frac35+\frac75}}{3^{\frac25+\frac35}}=\frac{2^2}{3^1}=\frac43$

g) $\sqrt[3]{4\cdot\sqrt[4]{8}}=\left(2^2\cdot 2^{\frac34}\right)^{\frac13}=\left(2^{\frac{11}{4}}\right)^{\frac13}=2^{\frac{11}{12}}=\sqrt[12]{2^{11}}=\sqrt[12]{2048}$

h) $\sqrt[5]{9\cdot\sqrt[3]{3\cdot\sqrt{27}}}=\left(3^2\cdot(3\cdot\sqrt{3^3})^{\frac13}\right)^{\frac15}=\left(3^2\cdot(3^{\frac52})^{\frac13}\right)^{\frac15}=\left(3^{2+\frac56}\right)^{\frac15}=3^{\frac{17}{6}\cdot\frac15}=3^{\frac{17}{30}}=\sqrt[30]{3^{17}}$

i) $\sqrt[6]{6\cdot\sqrt[4]{6\cdot\sqrt[3]{6}}}=\left(6\cdot(6\cdot 6^{\frac13})^{\frac14}\right)^{\frac16}=\left(6\cdot(6^{\frac43})^{\frac14}\right)^{\frac16}=\left(6^{1+\frac13}\right)^{\frac16}=6^{\frac{4}{3\cdot 6}}=6^{\frac29}=\sqrt[9]{36}$

7. a) $\sqrt[4]{a^2\cdot\sqrt[3]{a^2\cdot\sqrt{a}}}=\left(a^2\cdot(a^{2+\frac12})^{\frac13}\right)^{\frac14}=\left(a^2\cdot a^{\frac56}\right)^{\frac14}=\left(a^{\frac{17}{6}}\right)^{\frac14}=a^{\frac{17}{24}}=\sqrt[24]{a^{17}}$

b) $\sqrt[3]{\frac{a^2}{b}\cdot\sqrt[2]{\frac{a^3}{b}}\cdot\sqrt[4]{\frac{b^3}{a^5}}}=\left(a^2\cdot b^{-1}\cdot a^{\frac32}\cdot b^{-\frac12}\cdot b^{\frac34}\cdot a^{-\frac54}\right)^{\frac13}=\left(a^{\frac94}\cdot b^{-\frac34}\right)^{\frac13}=a^{\frac34}\cdot b^{-\frac14}=\frac{\sqrt[4]{a^3}}{\sqrt[4]{b}}=\frac{\sqrt[4]{a^3b^3}}{b}$

c) $\sqrt[5]{c^4\cdot\sqrt[3]{c^2}}\cdot\sqrt{c\cdot\sqrt[4]{c^3}}:\sqrt[24]{c^{41}}=\left(c^4\cdot c^{\frac23}\right)^{\frac15}\cdot\left(c\cdot c^{\frac34}\right)^{\frac12}:c^{\frac{41}{24}}=c^{\frac{14}{3\cdot 5}}\cdot c^{\frac{7}{4\cdot 2}}\cdot c^{-\frac{41}{24}}=c^{\frac1{10}}=\sqrt[10]{c}$

d) $\sqrt[4]{a\cdot\sqrt[6]{ab^2}}\cdot\sqrt[3]{\sqrt[8]{a^{11}b^{-1}}}\cdot\sqrt{b^2\cdot\sqrt[4]{a^2}}=\left(a\cdot a^{\frac16}\cdot b^{\frac26}\right)^{\frac14}\cdot\left((a^{11}b^{-1})^{\frac18}\right)^{\frac13}\cdot\left(b^2\cdot a^{\frac24}\right)^{\frac12}=ab\cdot\sqrt[4]{a^3}\cdot\sqrt[24]{b}$