

Addition und Subtraktion von Bruchtermen * Jahrgangsstufe 8

Bringe auf den Hauptnenner, fasse zusammen und kürze, falls möglich!

1. $\frac{1}{a} - \frac{1}{b} + \frac{a+b}{ab}$

2. $\frac{4x-3y}{a+b} - \frac{6x-5y}{2a+2b} + \frac{-3+2y}{3a+3b}$

3. $\frac{4p-7q}{7m-4n} + \frac{p+5q}{-7m-4n} - \frac{3p+q}{7m+4n}$

4. $\frac{1}{4p-7q} + \frac{49q}{21pq-12p^2} - \frac{8p+3q}{12pq-21q^2}$

5. $\frac{5s+7}{t^2-4t+4} - \frac{6s-7}{4-t^2}$

6. $\frac{2a^2-a+21}{12a^2-27} + \frac{2a-7}{6a-9} - 1$

7. $\frac{m^2}{m^2-4n^2} - \frac{7m+4n}{10m+20n} + \frac{1}{5}$

8. $5 - \frac{5x+9}{x+2} + \frac{8}{x^4-16}$

9. $\frac{16y^4+60y+9}{16y^4-81} + \frac{5}{4y^2+9} - 1$

10. $\frac{6x+1}{4x-6} + \frac{8x-3}{6x+9} - \frac{20x^2-4x+9}{8x^2-18}$

11. $\frac{6x+1}{4x-6} + \frac{8x-3}{6x+9} - \frac{20x^2-4x+9}{8x^2-18}$

12. $\frac{3a-b}{3x-6y} + \frac{4a-b}{-4x+8y} + \frac{b(x+y)}{2x^2-8y^2}$

13. $\frac{2x^2-27x-49}{735-15x^2} - \frac{-2x+4}{3x-21} + \frac{4x-1}{5x+35}$

14. $\frac{a^4+1}{a^4-1} + \frac{a^2+1}{a^2-1} - \frac{a+1}{a-1}$

15. $\frac{3ab}{9a^2-b^2} + \frac{a+b}{2b-6a} + \frac{3a^2-4ab-b^2}{18a^2-2b^2}$

16. $\frac{2}{x+1} - \frac{x-3}{x^2-1} - \frac{x}{x^2-2x+1}$



Lösungen:

$$1. \frac{1}{a} - \frac{1}{b} + \frac{a+b}{ab} = \frac{1 \cdot b}{a \cdot b} - \frac{1 \cdot a}{b \cdot a} + \frac{a+b}{ab} = \frac{b-a+a+b}{ab} = \frac{2b}{ab} = \frac{2}{a}$$

$$2. \frac{4x-3y}{a+b} - \frac{6x-5y}{2a+2b} + \frac{-3+2y}{3a+3b} = \frac{(4x-3y) \cdot 6}{(a+b) \cdot 6} - \frac{(6x-5y) \cdot 3}{2(a+b) \cdot 3} + \frac{(-3+2y) \cdot 2}{3(a+b) \cdot 2} =$$
$$\frac{24x-18y-18x+15y-6+4y}{6(a+b)} = \frac{6x+y-6}{6(a+b)}$$

$$3. \frac{4p-7q}{7m-4n} + \frac{p+5q}{-7m-4n} - \frac{3p+q}{7m+4n}$$
$$= \frac{(4p-7q) \cdot (7m+4n)}{(7m-4n) \cdot (7m+4n)} + \frac{(p+5q) \cdot (7m-4n)}{-(7m+4n) \cdot (7m-4n)} - \frac{(3p+q) \cdot (7m-4n)}{(7m+4n) \cdot (7m-4n)}$$
$$= \frac{28pm+16pn-49qm-28qn}{(7m-4n) \cdot (7m+4n)} - \frac{(7pm-4pn+35qm-20qn)}{(7m+4n) \cdot (7m-4n)} - \frac{(21pm-12pn+7qm-4qn)}{(7m+4n) \cdot (7m-4n)}$$
$$= \frac{28pm+16pn-49qm-28qn-7pm+4pn-35qm+20qn-21pm+12pn-7qm+4qn}{(7m-4n) \cdot (7m+4n)}$$
$$= \frac{32pn-91qm-4qn}{(7m-4n) \cdot (7m+4n)}$$

$$4. \frac{1}{4p-7q} + \frac{49q}{21pq-12p^2} - \frac{8p+3q}{12pq-21q^2} = \frac{1 \cdot 3pq}{(4p-7q) \cdot 3pq} + \frac{49q \cdot q}{-3p(-7q+4p) \cdot q} - \frac{(8p+3q) \cdot p}{3q(4p-7q) \cdot p}$$
$$= \frac{3pq}{3pq(4p-7q)} - \frac{49q^2}{3pq(4p-7q)} - \frac{8p^2+3pq}{3pq(4p-7q)} = \frac{3pq-49q^2-8p^2-3pq}{3pq(4p-7q)} = \frac{-49q^2-8p^2}{3pq(4p-7q)}$$

$$5. \frac{5s+7}{t^2-4t+4} - \frac{6s-7}{4-t^2} = \frac{5s+7}{(t-2)^2} - \frac{6s-7}{(2-t) \cdot (2+t)} = \frac{5s+7}{(t-2)^2} + \frac{6s-7}{(t-2) \cdot (2+t)}$$
$$= \frac{(5s+7) \cdot (2+t)}{(t-2)^2 \cdot (2+t)} + \frac{(6s-7) \cdot (t-2)}{(t-2) \cdot (2+t) \cdot (t-2)} = \frac{10s+5st+14+7t+6st-12s-7t+14}{(t-2)^2 \cdot (2+t)} = \frac{-2s+11st+28}{(t-2)^2 \cdot (2+t)}$$

$$6. \frac{2a^2-a+21}{12a^2-27} + \frac{2a-7}{6a-9} - 1 = \frac{2a^2-a+21}{3(2a-3)(2a+3)} + \frac{(2a-7) \cdot (2a+3)}{3(2a-3) \cdot (2a+3)} - \frac{3(2a-3) \cdot (2a+3)}{3(2a-3) \cdot (2a+3)}$$
$$= \frac{2a^2-a+21+4a^2+6a-14a-21-(12a^2-27)}{3(2a-3)(2a+3)} = \frac{-6a^2-9a+27}{3(2a-3)(2a+3)} = \frac{3(-2a^2-3a+9)}{3(2a-3)(2a+3)}$$
$$= \frac{-2a^2-3a+9}{(2a-3)(2a+3)}$$

$$7. \frac{m^2}{m^2-4n^2} - \frac{7m+4n}{10m+20n} + \frac{1}{5}$$
$$= \frac{m^2 \cdot 10}{(m-2n)(m+2n) \cdot 10} - \frac{(7m+4n) \cdot (m-2n)}{10(m+2n) \cdot (m-2n)} + \frac{1 \cdot 2(m+2n)(m-2n)}{5 \cdot 2(m+2n)(m-2n)}$$
$$= \frac{10m^2 - (7m^2 - 14mn + 4mn - 8n^2) + 2m^2 - 8n^2}{10(m-2n)(m+2n)} = \frac{5m^2 + 10mn}{10(m-2n)(m+2n)}$$
$$= \frac{5m(m+2n)}{10(m-2n)(m+2n)} = \frac{m}{2(m-2n)}$$

$$\begin{aligned}
 8. \quad & 5 - \frac{5x+9}{x+2} + \frac{8}{x^4-16} = \frac{5 \cdot (x^4-16)}{x^4-16} - \frac{(5x+9) \cdot (x-2) \cdot (x^2+4)}{(x+2) \cdot (x-2) \cdot (x^2+4)} + \frac{8}{x^4-16} \\
 & = \frac{5x^4 - 80 - (5x^4 - x^3 + 2x^2 - 4x - 72) + 8}{x^4-16} = \frac{x^3 - 2x^2 + 4x}{x^4-16} = \frac{x(x^2 - 2x + 4)}{x^4-16} \\
 & = \frac{x(x-2)^2}{(x-2)(x+2)(x^2+4)} = \frac{x(x-2)}{(x+2)(x^2+4)}
 \end{aligned}$$

9.

$$\begin{aligned}
 & \frac{16y^4 + 60y + 9}{16y^4 - 81} + \frac{5}{4y^2 + 9} - 1 = \frac{16y^4 + 60y + 9}{(4y^2 - 9)(4y^2 + 9)} + \frac{5 \cdot (4y^2 - 9)}{(4y^2 + 9) \cdot (4y^2 - 9)} - \frac{(4y^2 - 9)(4y^2 + 9)}{(4y^2 - 9)(4y^2 + 9)} \\
 & = \frac{16y^4 + 60y + 9 + 20y^2 - 45 - (16y^4 - 81)}{(4y^2 - 9)(4y^2 + 9)} = \frac{60y + 45 + 20y^2}{(4y^2 - 9)(4y^2 + 9)} = \frac{5 \cdot (4y^2 + 12y + 9)}{(2y - 3)(2y + 3)(4y^2 + 9)} \\
 & = \frac{5 \cdot (4y^2 + 12y + 9)}{(2y - 3)(2y + 3)(4y^2 + 9)} = \frac{5 \cdot (2y + 3)^2}{(2y - 3)(2y + 3)(4y^2 + 9)} = \frac{5 \cdot (2y + 3)}{(2y - 3)(4y^2 + 9)}
 \end{aligned}$$

10.

$$\begin{aligned}
 & \frac{8x^2 - 0,5a}{2x^2 - ax} - \frac{7x + 0,5a - 1}{2x - a} + \frac{x - 1}{2x} = \frac{(8x^2 - 0,5a) \cdot 2}{x(2x - a) \cdot 2} - \frac{(7x + 0,5a - 1) \cdot 2x}{(2x - a) \cdot 2x} + \frac{(x - 1) \cdot (2x - a)}{2x \cdot (2x - a)} \\
 & = \frac{16x^2 - a - (14x^2 + ax - 2x) + 2x^2 - ax - 2x + a}{2x(2x - a)} = \frac{4x^2 - 2ax}{2x(2x - a)} = \frac{2x \cdot (2x - a)}{2x \cdot (2x - a)} = 1
 \end{aligned}$$

11.

$$\begin{aligned}
 & \frac{6x+1}{4x-6} + \frac{8x-3}{6x+9} - \frac{20x^2-4x+9}{8x^2-18} = \frac{(6x+1) \cdot 3(2x+3)}{2(2x-3) \cdot 3(2x+3)} + \frac{(8x-3) \cdot 2(2x-3)}{3(2x+3) \cdot 2(2x-3)} - \frac{(20x^2-4x+9) \cdot 3}{2(2x+3)(2x-3) \cdot 3} \\
 & = \frac{36x^2 + 54x + 6x + 9 + 32x^2 - 48x - 12x + 18 - (60x^2 - 12x + 27)}{2(2x-3) \cdot 3(2x+3)} = \frac{8x^2 + 12x}{2(2x-3) \cdot 3(2x+3)} \\
 & = \frac{8x^2 + 12x}{2(2x-3) \cdot 3(2x+3)} = \frac{2 \cdot 2x \cdot (2x+3)}{2(2x-3) \cdot 3(2x+3)} = \frac{2x}{3(2x-3)}
 \end{aligned}$$

12.

$$\begin{aligned}
 & \frac{3a-b}{3x-6y} + \frac{4a-b}{-4x+8y} + \frac{b(x+y)}{2x^2-8y^2} = \frac{(3a-b) \cdot 4(x+2y)}{3(x-2y) \cdot 4(x+2y)} + \frac{(4a-b) \cdot 3(x+2y)}{-4(x-2y) \cdot 3(x+2y)} + \frac{b(x+y) \cdot 6}{2(x-2y)(x+2y) \cdot 6} \\
 & = \frac{12ax + 24ay - 4bx - 8by}{12(x-2y)(x+2y)} - \frac{12ax + 24ay - 3bx - 6by}{4(x-2y) \cdot 3(x+2y)} + \frac{6bx + 6by}{12(x-2y)(x+2y)} = \frac{5bx + 4by}{12(x-2y)(x+2y)}
 \end{aligned}$$

$$13. \quad \frac{2x^2 - 27x - 49}{735 - 15x^2} - \frac{-2x + 4}{3x - 21} + \frac{4x - 1}{5x + 35} = \frac{10x + 70}{5 \cdot (7+x)(7-x)} = \frac{2}{7-x}$$

14.

$$\frac{a^4+1}{a^4-1} + \frac{a^2+1}{a^2-1} - \frac{a+1}{a-1} = \frac{a^4+1}{a^4-1} + \frac{(a^2+1) \cdot (a^2+1)}{(a^2-1) \cdot (a^2+1)} - \frac{(a+1) \cdot (a+1)(a^2+1)}{(a-1) \cdot (a+1)(a^2+1)} = \frac{a^4 - 2a^3 - 2a + 1}{(a^2+1)(a+1)(a-1)}$$

15.

$$\begin{aligned}
 & \frac{3ab}{9a^2 - b^2} + \frac{a+b}{2b-6a} + \frac{3a^2 - 4ab - b^2}{18a^2 - 2b^2} = \frac{3ab \cdot 2}{(9a^2 - b^2) \cdot 2} + \frac{(a+b) \cdot (3a+b)}{-2(-b+3a) \cdot (3a+b)} + \frac{3a^2 - 4ab - b^2}{18a^2 - 2b^2} \\
 & = \frac{-2ab - 2b^2}{2(3a-b)(3a+b)} = \frac{-2b(a+b)}{2(3a-b)(3a+b)} \left[= \frac{-b(a+b)}{(3a-b)(3a+b)} = \frac{b(a+b)}{(-3a+b)(3a+b)} = \frac{ab+b^2}{b^2-9a^2} \right]
 \end{aligned}$$

16.

$$\frac{2}{x+1} - \frac{x-3}{x^2-1} - \frac{x}{x^2-2x+1} = \frac{2(x-1)^2 - (x-3) \cdot (x-1) - x \cdot (x+1)}{(x-1)^2(x+1)} = \frac{-(x+1)}{HN} = -\frac{1}{(x-1)^2}$$